Pathways to Calculus: A Research-Based Model for Transforming Precalculus-Level Mathematics Teaching and Learning

Background

Over the past 20 years, mathematicians and mathematics educators have engaged in numerous projects to re-conceptualize the teaching of precalculus-level mathematics so that students are more prepared to learn and succeed in calculus (e.g., Carlson, 1998; Carlson, Jacobs, Coe, Larsen & Hsu, 2002; Harel & Dubinsky, 1992; Monk, 1992; Thompson, 1995). Many of these projects have generated knowledge of the mathematical thinking that is needed to help students understand concepts such as function, rate of change, and exponential growth. Other projects have identified the mathematical habits of mind and problem-solving abilities that students need to acquire for success in STEM fields. This knowledge has not had a widespread impact on the teaching of undergraduate mathematics courses in the U.S., the result of which is (1) the teaching of precalculus courses are currently not effective in terms of student retention or in supporting student learning of key course ideas (e.g., Carlson & Oehrtman, 2012); and (2) typical approaches to teaching precalculus do not incorporate research-based knowledge on student learning to support improvements in the teaching and learning of undergraduate mathematics.

If one considers the processes of scientific inquiry that have led to new products and practices in engineering, medicine, business, etc., we observe very different processes at play in the practice of improving mathematics teaching and curriculum development. As a response, we have leveraged our and others' prior work on understanding and teaching key ideas of precalculus, and have engaged in multiple iterations of design and research to develop the Pathways Professional Development Model for Precalculus Level Mathematics (P^3DM) (*Project Pathways*: NSF Grants: DUE 0412537 & 1050721).

The Pathways resources include student curricula (e.g., in-class student activities, an online interactive textbook, online videos), teacher materials (e.g., teacher notes, exemplary lecture videos, and dynamic computer applets for use in instruction), and workshops designed to support teachers in engaging their students in genuine inquiry and mathematically substantive conversations. The Pathways Project instructor resources have been documented to be useful for helping teachers shift their instruction so that their students acquire both the understandings and mathematical practices articulated in the *Common Core Standards for Mathematics*. The student resources are organized around key ideas of precalculus mathematics and are packaged in 12 modules that can be accessed online. The summer workshops engage precalculus instructors in completing the in-class activities and discussing the lesson's rationale and implementation. The detailed teacher notes for the in-class activities and homework provide model solutions and explanations, and description of common student misconceptions. A sample introductory trigonometry lesson with teacher notes can be viewed at https://www.rationalreasoning.net/products.php

A Lesson for Introducing the Concept of Function

In an introductory lesson on functions, students are supported in conceptualizing the quantities in a problem context and developing a meaningful formula that describes how two quantities change together. Studies of student thinking on a collection of tasks have revealed that students who effectively conceptualize the varying and fixed quantities in an applied problem are able to

build meaningful formulas, graphs, and tables, and then use these representations to describe how the quantities are changing together. This finding has influenced the design of Pathways tasks intended to support students in acquiring the reasoning abilities, approaches, and confidence to succeed in using precalculus concepts to solve novel "word problems."

Documented Results

The *Pathways Precalculus Professional Development Model* was designed, researched, and refined over an eight-year period, and is now consistently realizing significant pre-post gains in precalculus students' scores on the Precalculus Concept Assessment (PCA)[1], with student mean scores improving from ~9.5 to ~17, out of 25. Further analysis of PCA data revealed that schools with large percentages of students receiving free and reduced lunches are achieving the same or greater PCA gains as the most affluent schools in three districts. This finding is consistent with research (Treisman, 1992) that found that engaging minority students in thought-provoking, collaborative work on problems centering on fundamental concepts in mathematics and science levels the playing field for underrepresented populations.

Our research on teachers and teaching has revealed that high gains in teachers' mathematical understandings, reasoning abilities, and beliefs about teaching and learning, although necessary, are often not sufficient for transforming a teacher's instructional practices. We engaged teachers in graduate courses and workshops over a four-year period prior to developing the *Pathways Precalculus* student curriculum and instructor resources and measured only small gains in student learning and teachers' questioning and the conceptual focus of their lectures. This is because the school curriculum they were using did not support student understanding of key concepts or construction of mathematical practices, and teachers were unable to engage students in meaningful mathematical activity in the context of the school tasks and assignments. However, with conceptually coherent materials that embody research knowledge on student learning, we have consistently documented teachers making sense of student thinking and their making profound shifts in their ability to pose questions that support students in developing reasoning patterns that lead to their understanding key concepts.

Potential Opportunities

The P^3DM model has been successfully scaled to over 30 high schools and 4 colleges and universities, including over 300 teachers and 20,000 students. The P^3DM resources are ready to be disseminated to other schools and districts across the nation.

For More Information

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[1] The PCA is a 25-item instrument that has been validated to assess student readiness for Calculus. The highest PCA mean score for precalculus students at the end of the course was 10.4.