MATHEMATICS, THE COMMON CORE, AND LANGUAGE

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September 19, 2012
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Understanding Language | Language, Literacy, and Learning in the Content Areas

Work supported by NSF, Spencer Foundation, UCSC Senate grants, and Understanding Language Initiative supported by Carnegie and Gates Foundations.
Center for the Mathematics Education of Latinos/as
National Science Foundation Award No. ESI-0424983

CEMELA

TALK OVERVIEW

1. Summary of research evidence on:
   - -Effective approaches to instruction for ELs
   - -Effective approaches to mathematics instruction
2. Recommendations for math instruction:
   - -Aligned with math Common Core
   - -For students who are learning English (ELs), and/or developing literacy, and/or academic language
   - -Illustrate with examples from video clip
3. Small group discussion
   - -How implement these recommendations?

THREE THEMES

Effective math instruction connected to language & literacy:

#1. Access to mathematics *teaching for understanding*

#2. Engage students in CC "Eight *Mathematical Practices*" use reading, writing, talking, multiple representations

#3. Use *everyday language as a resource*

EFFECTIVE INSTRUCTION FOR ELS

Very quick summary of research!

- AERA Research Points (online):
  Closing the Gap-High Achievement for Students of Color.
- Garcia, E. & Gonzalez, R. (1995). Issues in systemic reform for culturally and linguistically diverse students. *Teachers College Record,* 96 (3), 418-431.
- Gándara, P. & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies.* Cambridge, MA: Harvard University Press.

EFFECTIVE INSTRUCTION FOR ELS

Research suggests that high-quality instruction for ELs that supports student achievement has two general characteristics:

- Treat language as a resource, not a deficit
- Emphasize academic achievement, not only learning English

Gándara & Contreras (2009)

EFFECTIVE TEACHING FOR ELS

Teachers documented as successful with students from non-dominant communities:

1. Reject models of their students as intellectually disadvantaged
2. Hold high expectations for all students
3. Have high commitment to students' academic success and to student-home communication
4. Can change curriculum and instruction to meet the specific needs of their students.

Garcia & Gonzalez (1995)

ACCESS TO IMPORTANT MATHEMATICS

1) Support programs that engage all students in a **rigorous, standards-based curriculum**. Provide additional time and instruction as needed, but **do not lower the expectations.**
2) Create environment that provides the necessary social supports for learning.

Closing the Gap: High Achievement for Students of Color.
AERA Research Points, Fall 2004 | Vol. 2, Issue 3.

EFFECTIVE INSTRUCTION FOR ELS

Characteristics of environments documented effective in supporting success for students from non-dominant communities:

- Curricula provide "abundant and diverse opportunities for speaking, listening, reading, and writing"
- Instruction "encourages students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts."

Garcia & Gonzalez (1995)

Opportunities for understanding, reasoning and sense making.

EFFECTIVE MATH INSTRUCTION

Resources that summarize research:

- AERA Research Points (online):
  Do the Math-Cognitive Demand Makes a Difference.
- Hiebert & Grouws (2007) The effects of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning,* NCTM.
  Short version (online): Effective Instruction Research Brief: Effective Teaching for the Development of Skill and Conceptual Understanding of Number: What is Most Effective?

HIGH COGNITIVE DEMAND TASKS

1) Embrace high expectations for all students in mathematics.
2) Institute curriculum policies that broaden course-taking options for traditionally underserved students. This includes avoiding systems of tracking students that limit their "opportunities to learn" and delay their exposure to college-preparatory mathematics coursework.

➢ **3. Raise cognitive demand in mathematics teaching and learning:**

Elevated thinking processes come into play when students focus on mathematical concepts and connections among those concepts.

High cognitive demand is reinforced when teachers maintain the rigor of mathematical tasks, for example, by encouraging students to explain their problem-solving.

Do the Math: Cognitive Demand Makes a Difference.
AERA Research Points, Fall 2006, Volume 4, Issue 2

CHARACTERISTICS OF EFFECTIVE MATH TEACHING

Mathematics teaching that makes a difference in student achievement and promotes conceptual development in mathematics has two central features:

1. Teachers and students attend explicitly to concepts, and
2. Teachers give students the time to wrestle with important mathematics.

Hiebert & Grouws (2007)

NCTM Research Brief online

Recommendations for Mathematics Instruction

- For ELs (and students developing literacy, reading, academic language, math discourse, etc.)

  "Important for all students, essential for ELs"

That:

- Aligns with Common Core State Standards
- Connects math content to language

ALIGN WITH CCSS?

FIRST: Teach math for understanding!

- ✓Students use and connect multiple representations
- ✓Students share and refine their reasoning
- ✓Students develop meaning for symbols

FREE resources online:

NCTM, Inside Mathematics, MARS, more…….

ALIGN WITH EMPHASES IN CCSS

Balance conceptual understanding & procedural fluency

Balance student activities addressing conceptual understanding and procedural fluency, connect two types of knowledge

Maintain high cognitive demand

Use and maintain high cognitive demand of math tasks in lessons and units

Develop beliefs

Support students in developing beliefs that math is sensible, worthwhile, and doable

www.corestandards.org/the-standards/mathematics

- ➢**Engage students in math practices (8)**

MATHEMATICAL PRACTICES

1) Make sense of problems and persevere in solving them
2) Reason abstractly and quantitatively
3) Construct viable arguments and critique the reasoning of others
4) Model with mathematics
5) Use appropriate tools strategically
6) Attend to precision
7) Look for and make use of structure
8) Look for and express regularity in repeated reasoning

www.corestandards.org/the-standards/mathematics/introduction/standards-for-mathematical-practice/

TO SUPPORT MATHEMATICAL PRACTICES

Engage students in solving problems and:

- Using and connecting multiple representations
- Looking for patterns
- Abstracting
- Generalizing
- Imagining

TO SUPPORT MATHEMATICAL PRACTICES

Engage students in solving problems:

- Using and connecting multiple representations
- Looking for patterns, abstracting, generalizing, imagining
- Communicating their thinking, conjecturing, justifying, convincing, proving
- Constructing and critiquing arguments
- Using special cases, counter examples, etc.
- Connecting claims to mathematical representations
- Attending to precision: Apply claims to precisely defined situations

TO SUPPORT MATHEMATICAL DISCUSSIONS

Engage students in:

- Communicating their thinking, conjecturing, justifying, convincing, proving,
- Constructing and critiquing arguments
- Using special cases, counter examples, etc.
- Connecting claims to mathematical representations
- Attending to precision: Apply claims to precisely defined situations

Instruction needs to include:

- Mathematical discussions
- MULTIPLE ways for students to participate:

  teacher-led discussions, small group exploration and discussion, pairs, student presentations, etc.

CAUTION!!!

To support mathematical discussion (discourse)

Instruction must include

MULTIPLE REPRESENTATIONS

Not only talk and text!

Also representations such as

Objects, manipulatives, drawings, symbols, equations, tables, graphs etc.

NCTM: "connecting multiple representations"

Recommendations for Effective Mathematics Instruction that Connects Math Content to Language

CLASSROOM VIGNETTE

"They never get together"

(Moschkovich, 1999)

This work was supported by Grants from NSF.
The Math Discourse Project at Arizona State University videotaped this lesson with support by an NSF grant.

CLASSROM VIGNETTE: "They never get together"

- 3rd grade Bilingual classroom, urban, California
- 33 students LEP (Limited English Proficiency)
- Lesson: ESL mathematics lesson where students use English to describe quadrilaterals using paper shapes

CLASSROM VIGNETTE: "They never get together"

Teacher introduced students to topics in Spanish

then later conducted lessons in English.

Students had been working on a unit on two-dimensional geometric figures.

For several weeks, instruction included vocabulary such as "radius," "diameter," "congruent," "hypotenuse" and names of different quadrilaterals in both Spanish and English. Students had been talking about shapes and teacher had asked them to point, touch, and identify different shapes.

TAKE A FEW MINUTES TO DESCRIBE A RECTANGLE

The teacher asked: "Who can describe a rectangle?"

Take a few minutes to describe a rectangle:

- Share some descriptions with person next to you.
- Or if you prefer working alone, write some descriptions down.

WATCH VIDEO #1
FOCUS QUESTIONS: STUDENTS

Focus on students:

1.How did students use language to communicate mathematical ideas?

2. What mathematical practices did students use?

3. What resources did students use?

Listen carefully
Watch Julian
Read subtitles
Read transcript after watching video

TRANSCRIPT

1. Teacher:

   Today we are going to have a very special lesson in which you really gonna have to listen. You're going to put on your best, best listening ears because I'm only going to speak in English. Nothing else. Only English. Let's see how much we remembered from Monday. Hold up your rectangles . . . high as you can. (Students hold up rectangles) Good, now. Who can describe a rectangle? Eric, can you describe it [a rectangle]? Can you tell me about it?

TRANSCRIPT

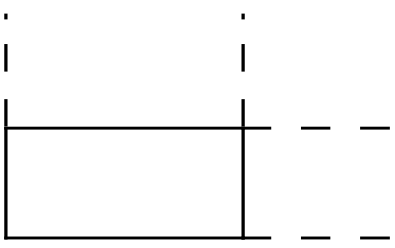
2. Eric: A rectangle has . . . two . . . short sides, and two . . . long sides.
3. Tcher: Two short sides and two long sides. **Can somebody tell me something else about this rectangle,** if somebody didn't know what it looked like, what, what . . . how would you say it.
4. Julian: **Parallela ……..**[holding up a rectangle, voice trails off].
5. Tcher: **It's parallel. Very interesting word. Parallel. Wow! Pretty interesting word, isn't it? Parallel. Can you describe what that is?**
6. Julian: **Never get together. They never get together** [runs finger over top side of rectangle].
7. Tcher: **What never gets together?**
8. Julian: **The parallela . . . they . . . when they go, they go higher, they never get together.** [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines]
9. Antonio: Yeah!
10. Tcher: **Very interesting. The rectangle then has sides that will never meet. Those sides will be parallel.** Good work. Excellent work.

NOTICE:
HOW JULIAN DESCRIBED A RECTANGLE

Julian:

The parallela . . . they . . . when they go, they go higher, they never get together

[runs two fingers parallel to each other first along the top and base of the rectangle and then continued along those lines].



NOTICE: MATHEMATICAL IDEAS

Parallel lines:

Straight lines in a plane which have no common point no matter how far they are extended. (Textbook definition)

Properties and categories

Property: "A rectangle has parallel sides"

Category: "A rectangle is a parallelogram"

During classroom discussions

- Students will not sound like a textbook definition!
- Emerging language and math ideas may be imperfect
- Focus on uncovering, hearing, supporting, and extending students 'mathematical reasoning

RECOMMENDATION #1

Focus on students' mathematical reasoning, not language accuracy.

#1. Focus on students' mathematical reasoning, not language accuracy

When the goal is to support students to:

- Participate in mathematical discussions
- Use mathematical reasoning
- Use emerging language

Less important:

Accuracy Clarity Single words

More important:

- ✓ Meaning of whole utterances
- ✓ Listening to students mathematical reasoning
- ✓ Uncovering the math in what they are saying

Focusing on mathematical reasoning can be challenging

Emerging language is imperfect (emerging ideas also)

- Julian's utterances are difficult to hear and interpret.
  He said the word "parallela" with hesitation.
- Mixed English and Spanish:
  Added "a" to parallel pronounced in Spanish
- A singular word "parallela" followed by plural "when they go higher"

Focusing on mathematical reasoning

Emerging language is imperfect (emerging ideas also)

- Julian's utterances are difficult to hear and interpret.
- Mixed English and Spanish.
- A singular word "parallela" followed by a plural "when they go higher."

Was his hesitation due to pronunciation, word choice, or math idea?

Impossible to answer this question!

- Julian is, in fact, accurately describing a property of parallel lines.
- If we focus only on language accuracy, we miss his mathematical reasoning.

Focusing on mathematical reasoning

Emerging language is imperfect
(emerging math ideas are also imperfect)

- Student utterances may be difficult to interpret
- Students may mix English and Spanish

➢ If we focus only on language accuracy, we can miss students' mathematical reasoning and sense making.

#1. Focus on students' mathematical reasoning, not language accuracy

In the Vignette……

Teacher did *not* focus directly on language accuracy, did not correct English, instead:

- Focused on the math content of what Julian said, not accuracy or clarity in how he said it.
- Moved past unclear or imperfect language: Moved past Julian's unclear utterance and use of the word "parallela."

WATCH VIDEO #2
FOCUS QUESTIONS: TEACHER

Focus on teaching:

1. What was the teacher doing?
2. How was the teacher supporting a mathematical discussion?

(We are not critiquing the teaching!)

Listen carefully

Read subtitles

Read transcript after watching video

RECOMMENDATION #2

Focus on math discourse practices, not "language" as single words, vocabulary, or grammar.

In the Vignette: What was teacher doing?

- Did not focus on single words or vocabulary

Instead:

- Asked questions to probe what Julian meant.
- Asked questions to uncover the math in what Julian said.
- Listened to students: tried to understand the math in what students said.
- Kept the discussion mathematical: focused on the math content of what students said and did.

#2. Focus on math practices,
not "language" as words, vocabulary, or grammar

Shift from simplified views of "language" as vocabulary, single words, grammar, or a list of definitions.

Why?

Over-emphasis on correct vocabulary and formal language limits the linguistic resources teachers and students can use to teach and learn mathematics with understanding.

#2. Focus on math practices, not single words or vocabulary

In the vignette:

What mathematical practices did Julian use?

Abstracting, generalizing, imagining

- He was describing an *abstract* property of parallel lines and making a *generalization* that parallel lines will *never* meet.

#2. Focus on math practices, not single words or vocabulary

In the vignette:

What mathematical practices did Julian use?

Abstracting, generalizing, imagining

- He was describing an *abstract* property of parallel lines and making a *generalization* that parallel lines will *never* meet.
- He was *imagining* what happens when the parallel sides of a rectangle are extended.

#2. Focus on math practices, not single words or vocabulary

In the vignette:

What mathematical practices did Julian use?

Abstracting, generalizing, imagining

- He was describing an *abstract* property of parallel lines and making a *generalization* that parallel lines will *never* meet.
- He was imagining what happens when the parallel sides of a rectangle are extended.

➢ If we only focused on accurate use of vocabulary, we would miss Julian's use of these mathematical practices.

CAUTION!!!

Mathematical Practice #6: Attend to precision

What does this mean?

Example: "Multiplication makes the result bigger"

Mathematical precision is not about a precise word:

- Making precise claims that apply only under particular constraints or conditions
- Knowing whether, when, and how precision in a calculation matters or not and if it does how

Recommendation #3

Recognize the complexity of language in math class.

More about #3 in the paper available online

Recommendation #3. Recognize complexity of language in math class

Language in math classrooms is complex

INCLUDES MULTIPLE:

a) Representations: objects, pictures, words, symbols, tables, graphs

b) Modes: oral, written, receptive, expressive

c) Kinds of written texts: textbooks, word problems, student explanations, teacher explanations

d) Kinds of talk: exploratory and expository (Barnes)

e) Audiences: presentations to teacher, peers, by teacher, by peers

Examples:
Complexity of language in math class

In the Vignette: What modes of expression did Julian and the teacher use?

- Gestures and objects, running his fingers along the parallel sides of a paper rectangle.
- The teacher used gestures and visual displays of geometric figures on the blackboard.

More in the paper online……

Recommendation #4

Treat everyday language as a resource, not an obstacle.

Recommendation #4. Treat everyday language as resource, not obstacle

Everyday and academic language are interdependent, related, not mutually exclusive.

There is no rigid separation between everyday language (out-of-school) and academic language (in-school).

#4. Everyday language as resource

In the vignette:

What language resources did Julian use to communicate his mathematical ideas?

- Julian used everyday expressions "go higher" and "get together"
- Everyday language was a resource not obstacle

#4. Everyday language as resource

In the vignette:

What language resources did Julian use to communicate his mathematical ideas?

- Julian used everyday expressions "go higher" and "get together"
- Resources not obstacles

What did teacher do?

✓ Provided formal words "extended" and "meet"

SUMMARY: Effective Math Instruction Connected to Language

#1. Focus on students' mathematical reasoning, not accuracy in using language

#2. Focus on mathematical discourse practices, not language as single words, vocabulary, or grammar

#3. Recognize the complexity of language in math classrooms (see paper online ell.stanford.edu/)

#4. Treat everyday language as a resource, not an obstacle

#5. Uncover the mathematics in what students say & do (see paper online ell.stanford.edu/)

SMALL GROUP DISCUSSION

TWO OPTIONS

Raise your hand:

1. Look at video again and discuss in small group
2. How can you implement these recommendations?

SMALL GROUP DISCUSSION

1. How can you implement these recommendations?

2. How can you support a shift from focusing on challenges and obstacles ELs face to resources and reasoning they use?

3. What are useful actions at different levels and settings:

Administrative

Teacher Professional Development, Coaching

Teaching, Tutoring, After School

Elementary, Middle, High School

THANK YOU!

- **Paper: "**Mathematics, the Common Core, and Language" Understanding Language: http://ell.stanford.edu/

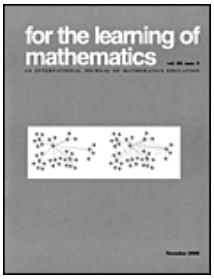
- **Publications** listed on my web page: http://people.ucsc.edu/~jmoschko/

SHORT ARTICLES USING THIS LESSON

Supporting the participation of English language learners in mathematical discussions.

1999, 19(1).

Examining mathematical Discourse practices, 2007, 27(1).



NCTM PUBLICATIONS

Using two languages when learning mathematics. NCTM Research Clip and Brief ONLINE

Supporting mathematical reasoning and sense making for English Learners. In Focus on High School Mathematics: Fostering Reasoning and Sense Making for All Students.