MINDSET

Mathematics INstruction using Decision Science and Engineering Tools

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NC STATE UNIVERSITY WAYNE STATE UNIVERSITY UNC CHARLOTTE

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Agenda

- MINDSET course overview
- What is Engineering Modeling
- Activities
  - Deterministic modeling – Optimizing profit in a LEGO production system
  - Probabilistic modeling -- Modeling the Space Shuttle Failure
- Project MINDSET results and contact information

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Description of MINDSET Project

- ***Engineering Modeling Mathematics course*** **based on Industrial Engineering and Operations Research tools**
- **Designed for juniors/seniors in high school**
- **Algebra II pre-requisite only, no calculus**
- **Intended for all high school students**

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Engineering and Science

What's the difference?

- Science is the study of ***natural*** artifacts
- Engineering is the design and creation of ***artificial*** artifacts in the service of mankind

Both the artifact and the process to make the artifact

Engineers must graduate from a nationally accredited engineering program and then pass two national licensing exams

source: *Engineering in K-12 Education,* NAE & NRC, National Academies Press, 2009, pg. 27

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Engineering and Technology

What's the difference?

“Technology” includes all types of human-made systems and processes — not in the limited sense often used in schools that equates technology with modern computational and communications devices

Technologies result when engineers apply their understanding of the natural world and of human behavior to design ways to satisfy human needs and wants (i.e., create human artifacts)

Source: *A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas*, National Research Council (NRC), The National Academies Press, 2012, p. 11-12

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Engineering Design, Industrial Design, Architectural Design, Textile Design, Art Design, …

What's the difference?

Engineering Design is focused on modeling behavior

Engineers want to understand what has happened and what will happen and this is done through models

- Drawing models – "ideation" sketches, hand sketches, CAD drawings, scale drawings, 3-D rotatable drawings, rough sketches, production drawings, etc.
- Physical models – static models, dynamic models, scale models, full size mockups, prototypes, proof of concepts, design testing, working and non-working, etc.
- Computational models – algorithms, static and dynamic simulations, animated simulations, etc., based on mathematical and descriptive relationships and constraints represented on a computer.
- Mathematical models – using mathematical relationships to represent physical behavior so as to *understand behavior* and to *predict behavior*

this is MINDSET!

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An *Engineering Modeling* Themed Math Course

Industrial Engineering tools are used to teach mathematics through modeling real world systems

These tools are primarily Operations Research techniques

Industrial Engineering focuses on people-based systems and not on mechanical, chemical or electrical devices.

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What's a model?

A *model* is an abstraction from reality that replicates the real behavior

Models have:

- Context

  Identifies the situation that is being modeled determining the model's boundaries and scope.

- Viewpoint

  The viewpoint determines through whose eyes the system is to be seen. It states the modeler's position as an observer or participant in the system.

- Purpose

  Establishes the objective to be achieved with the model.

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Engineering Modeling in the Real World

MINDSET

(Mathematics INstruction using Decision Science and Engineering Tools)
Industrial Engineering, Operations Research and Mathematics Education working together

Deterministic Modeling

(Volume I)

Probabilistic Modeling

(Volume II)

Indirect Content

- Excel knowledge (sufficient to take the Microsoft Excel level 1 certification)
- Computational Thinking used extensively in every chapter

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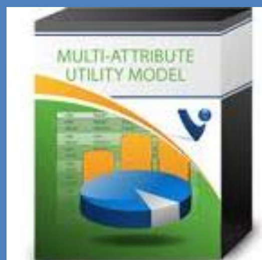
Deterministic Modeling

(Non Random Math Models)
Volume I

Telephone Plans
MCDM
(Chp. I)



Criteria

Measure

Weights

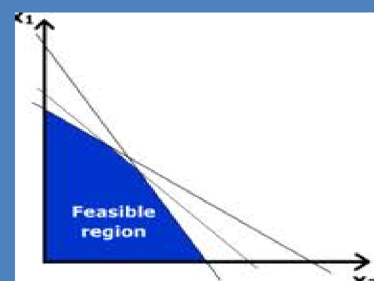
Results Interpretation

Sport Shoe
Sensitivity
(Chp. 3)

Furniture Manufacturing
Maximization
(Chp. 2)

Watershed Runoff
Minimization
(Chp. 4)

TV Advertising
Integer Prog.
(Chp. 5)



College App.
Binary Prog.
(Chp. 6)

Smoothie Co.
Location
(Chp. 7)

Linear Programming

Decision Variables

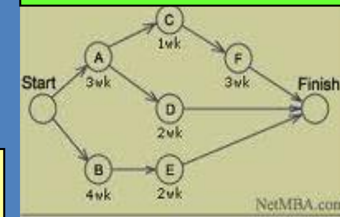
Objective Function

Constraints

Rumors
Shortest Path
(Chp. 8)

Getting Ready for School
Critical Path
(Chp. 9)

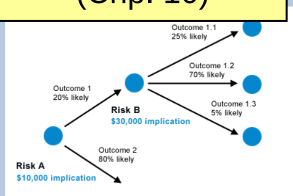
Network Theory



Probabilistic Modeling

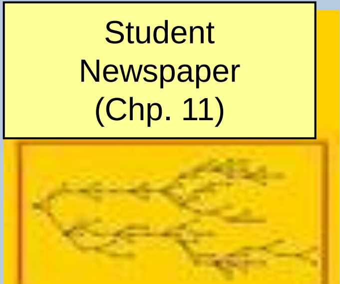
(Random Math Models)
Volume II

Auto Insurance
(Chp. 10)



Decision Trees

Student Newspaper
(Chp. 11)



Intro to Probability

False Positives / Negatives
(Chp. 12)

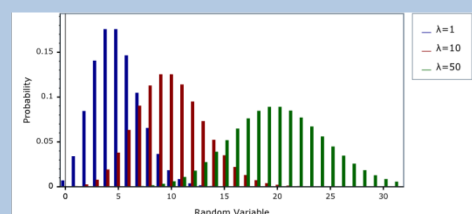
School Newspaper Publishing
(Chp. 13)

Binomial

Geometric

CSI Hiring
(Chp. 14)

Poisson



Toy Store Sales
(Chp. 15)

Normal

Quality Control
(Chp. 16)

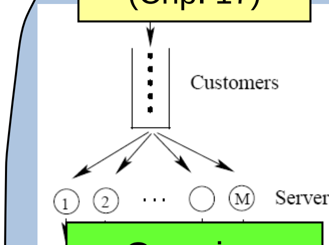
Using the Normal

Distributions

Random Variables

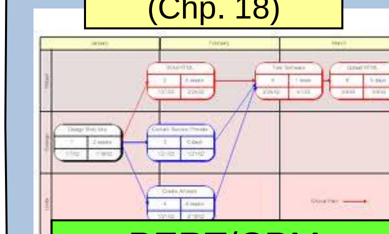
Independent Repetitions

Event Tickets
(Chp. 17)



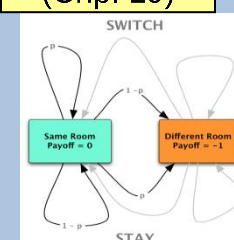
Queuing Theory

Family Dinner
(Chp. 18)



PERT/CPM Network Theory

Cholesterol
(Chp. 19)



Markov Decisions

STRUCTURE OF CHAPTER

- 3 “real-life” situation problems
- First Problem - introduction
    - Purpose of a particular problem-solving technique
- Second Problem – context
    - Explains every step in process
    - Interprets solutions
- Third Problem – synthesis
    - Presents solution
    - Students are expected to interpret results in the context of problem
- Still learning how teachers are using the unconventional design of the textbook’s chapters

MINDSET Curriculum and Textbook

When will we ever use this?

Making Decisions Using Advanced Mathematics

Volume I: Deterministic Modeling

1. Multi-Criteria Decision Making
2. Linear Programming -- Max
3. Sensitivity Analysis
4. Linear Programming -- Min
5. Integer Programming
6. Binary Programming
7. Location Problems
8. Min Spanning Trees & Shortest Path
9. Project Planning -- Critical Path Method (CPM)

Volume II: Probabilistic Modeling

10. Decision Trees
11. Intro to Probabilistic Modeling
12. Detecting & Interpreting False Positive & False Negative Results
13. Binomial & Geometric Distributions
14. Poisson Distribution
15. Normal Distribution
16. Quality Control
17. Queuing Theory
18. Project Planning – Program Evaluation Review Technique (PERT)
19. Markov Chains

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A Deterministic Example: The Lego Problem

Making decisions using deterministic (algebraic) models of the situation

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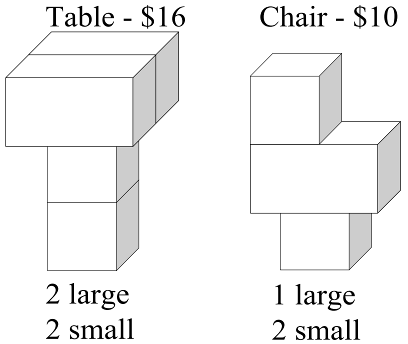
The LEGO problem
-- Optimizing Profit in Production --

A furniture company assembles tables and chairs from two different sized LEGOs. We can decide how many tables and chairs to make as the mix of large and small LEGOs changes we want to know:

- How would the production rate change?
- What would be the optimal profit?



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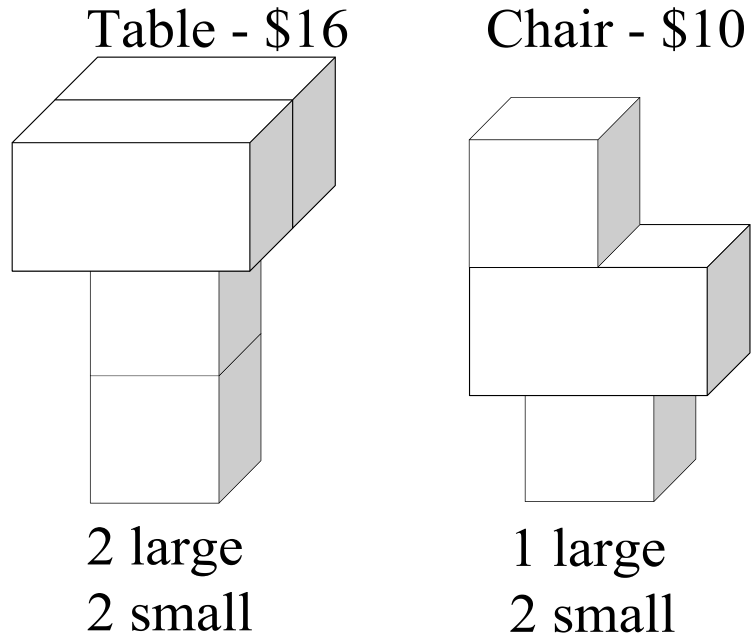
The Lego Problem

- Adapted from “Lego of My Simplex” by Norman Pendegraft from OR/MS Today (Feb 1997)
- Aligns with CCSS
  - The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions

2.0 Lego Activity

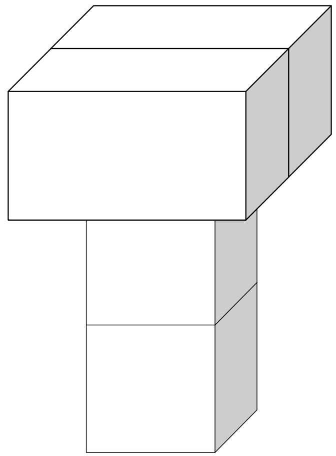
- If the furniture company obtains 6 large and 8 small pieces every day, what production generates the most profit?
- What are assumptions?



2.0 Lego Activity

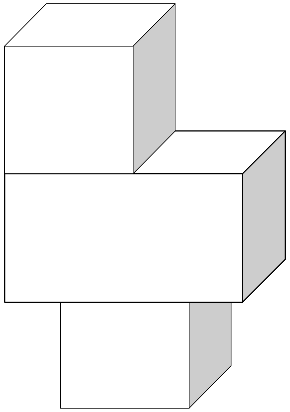
- Create a way to show your answer of maximizing profit.

Table - $16



2 large
2 small

Chair - $10



1 large
2 small

2.0 Lego Activity (continued)

- What if *nine small* pieces are available instead of eight? (there are still six large pieces)
  - How would the production rate change?
  - What would be the optimal profit?
- What if *seven large* pieces are available instead of six? (there are still eight small pieces)
- What if *seven large* pieces and *nine small* pieces are available?

Lego Activity: Formulation

- Going back to the original problem

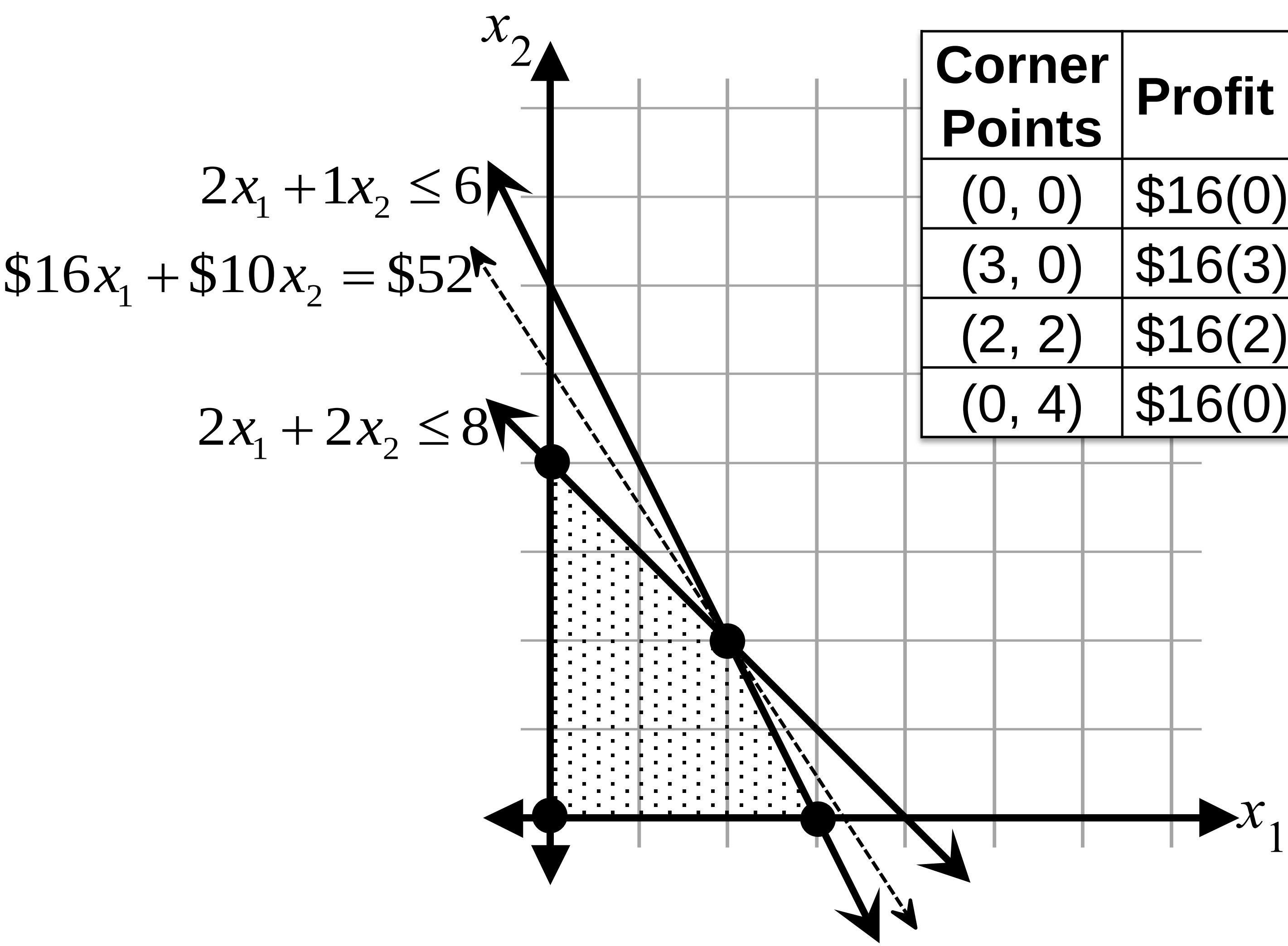
- Decision Variables
  - x_1 = the number of *tables* produced in a day
  - x_2 = the number of *chairs* produced in a day

- Objective Function
  - Maximize: $z = \$16x_1 + \$10x_2$

Lego Activity: Formulation (continued)

- Constraints
  - Maximum number of Large Blocks
    - $2x_1 + 1x_2 \leq 6$
  - Maximum number of Small Blocks
    - $2x_1 + 2x_2 \leq 8$
  - Non-negativity
    - $x_1, x_2 \geq 0$

2.0 Lego Activity: Formulation (continued)



Corner Points	Profit
(0, 0)	\$16(0) + \$10(0) = \$0
(3, 0)	\$16(3) + \$10(0) = \$48
(2, 2)	\$16(2) + \$10(2) = \$52
(0, 4)	\$16(0) + \$10(4) = \$40

2.0 Lego Problem on Excel

	A	B	C	D	E	F
1	Pre-Activity					
2	Lego Activity					
3	Profit Maximization					
4						
5	Decision Variable	**Tables** (x_1)	**Chairs** (x_2)			
6	Decision Values [# to make per day]	**2**	**2**			
7						**Total Profit**
8	Objective Function [Profit ($)]	16	10			**$52**
9						
10	**Constraints**			**Used**		**Available**
11	Maximum # of Large Blocks	2	1	**6**	≤	6
12	Maximum # of Small Blocks	2	2	**8**	≤	8

A Probabilistic Example: The NASA problem

Making decisions using probabilistic models of the situation

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Modeling the Probability of a Space Shuttle Failure

NASA engineers estimated a 1 in 80 chance of a catastropic Space Shuttle failure. Given this probability of failure, does it seem reasonable that there were 2 catastrophic failures in 135 shuttle missions?

Data & problem validated by Dr. Nancy Currie, Principal Engineer, NASA Engineering & Safety Center, and Former Space Shuttle Astronaut and Colonel (U.S. Army retired)

Challenger Disaster video on youtube

http://www.youtube.com/watch?v=ljPYmSdyVZc

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NASA Shuttle Catastrophic Failure

- The NASA team of engineers and managers collectively estimated that there was a 1 in 80 chance of catastrophic failure
- They anticipated flying the shuttle only 50 times, but by the end of 2011, there were 135 shuttle missions, two of which ended in catastrophe
  - The final space shuttle mission ended on July 21, 2011
- Q1. Given the estimated probability of a catastrophic failure is 1/80, does it seem reasonable that there were 2 catastrophic failures in 135 shuttle missions?

NASA Shuttle Catastrophic Failure

- For this context, we will consider the **Geometric Distribution**, which describes the probability of the first time a "success" occurs
  - What represents a "success" in this problem? What represents a "failure"?

NASA Shuttle Catastrophic Failure

- The NASA team of engineers and managers collectively estimated that there was a 1-in-80 chance of catastrophic failure

Look at NASA Shuttle Theoretically

- Using the 1-in-80 estimation:
  - Q2. What is the probability that there will be a catastrophe on the first flight?
  - Q3. What is the probability that the first five flights are safe and there is a catastrophe on the sixth flight?
  - Q4. What is the probability that the first catastrophe occurs on the 25^{th} flight?
  - Q5. What is the probability that the first catastrophe occurs on the 50^{th} flight?
  - Q6. What is the probability that there is a safe flight the first n – 1 flights and catastrophe on the nth flight?

Geometric Distribution: Theoretical

- If the first success occurs on trial k, then all of $(k - 1)$ trials before trial k must have been failures
- If the probability of a success is p, then the probability of a failure must be $1 - p$
- The formula that will give us the probability that the first success occurs on trial k

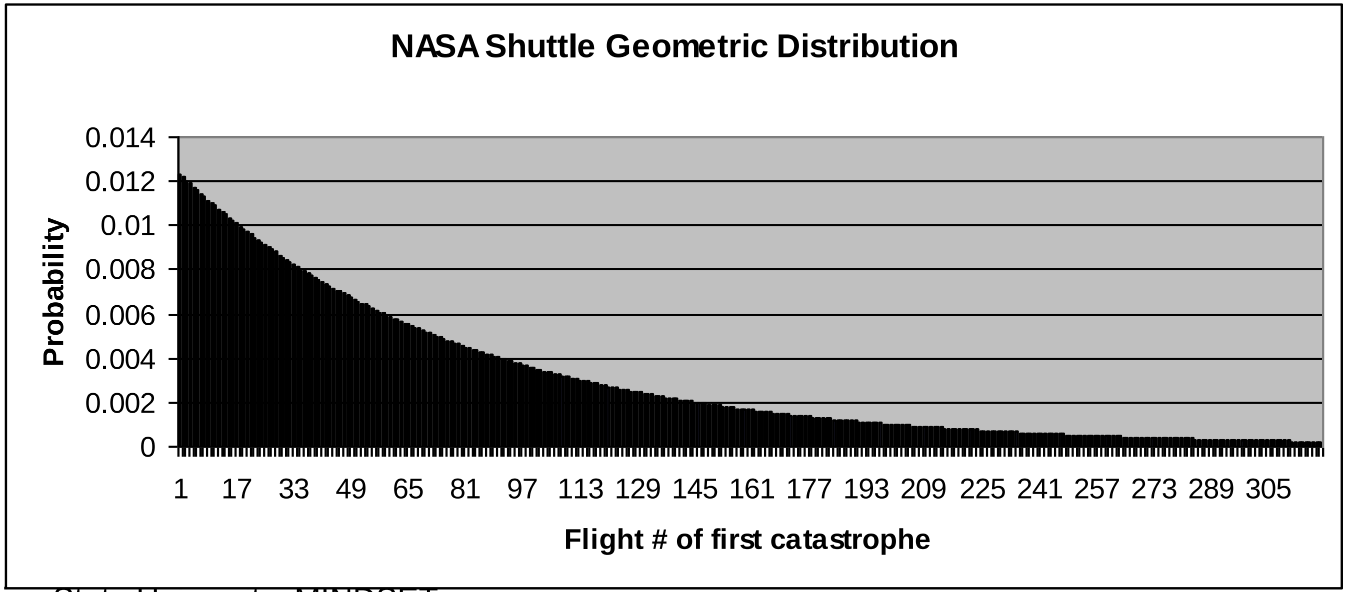
$$P(k) = (1 - p)^{k-1} \cdot p$$

Interpretation

- Q11. Now that there is further investigation of the NASA shuttle missions, does it seem reasonable that there were 2 catastrophic failures in 135 shuttle missions given the estimated probability of a catastrophic failure is 1/80?

Geometric Distribution: Excel

- In Excel, to find the probability that the first success occurs on trial *k*, where the probability of a success is *p*, type in =NEGBINOMDIST(*k*, 1, *p*)
- Note: This is a discrete graph

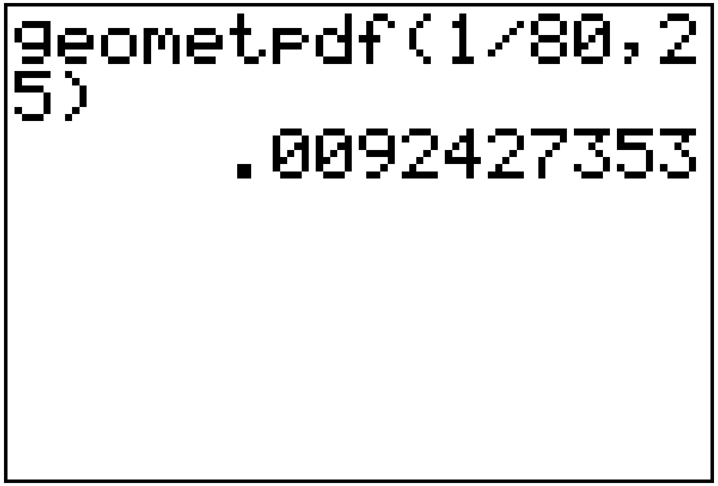


Geometric Distribution: Calculator

- In the graphing calculator, to find the probability that the first success occurs on trial x, where the probability of a success is p, go to the Distributions menu and type in geometpdf(p, x)

Geometric Distribution: Calculator

- For example, when p = 1/80 and x = 25, geometpdf(1/80, 25) = 0.009. What does this mean in terms of the problem?



NASA Shuttle: Going Further

- What is the probability that the first catastrophe occurs on or before the 25th flight?
    - Approximately 0.2698
- In the calculator, go to the Distributions menu and type in geometcdf(*p*, *x*)

Some Implementation Details

- 14 summer one and two week summer workshops to train approximately 250 high school teachers in Michigan, North Carolina, and Georgia
- The course in Georgia is Advanced Decision Making in Industry and Government **MIG**
- Graduate student onsite visits and other help when requested
- Teacher support through website, telephone and email system, social networking site
- Website delivers all current versions of curriculum, tests, solutions to all the questions, PowerPoint's for teaching, etc.

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NASA Shuttle

- What decision would we use this new geometric distribution to make?

- Questions and comments (if time)

MINDSET Georgia Contact Information

- Online Teacher Education Course

  Mathematics of Industry and Government

  Mr. Douglas Edwards

  CEISMC at Georgia Tech

  https://www.ceismc.gatech.edu/freeplucourses

- Georgia Dept. of Education

  Mr. Brook Kline, Lead Program Specialist

  Georgia Dept. of Education

  Bkline@doe.k12.ga.us

  (404) 657-9064

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Project MINDSET Results

Administration of Formal Assessment

Participants

	MINDSET (Project Group)	Non-MINDSET (Control Group)
Number of Students	272	110
Number of Teachers	11	4

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Project MINDSET Results

I. Comparison of Mindset (experimental) with Comparison (control) group, ANCOVA on posttest scores with pretest scores as covariate:
No significant difference in PreTest between groups.

II. Comparison of Mindset with Comparison group, Posttest only
For entire PSA, no statistically significant difference.
For items 1-8 PSA, no statistically significant difference.
For items 9-14 PSA, Mindset (mean - 17.68) statistically significantly higher than Comparison group (mean = 16.67).
(t = 4.94, df - 378.4, p = .00)

III. MINDSET group showed a statistically significant increase
6.1% increase
Effect Size = .13, Small Treatment effect

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Questions?

- Robert Young----- young@ncsu.edu
- Karen Keene------ karen_keene@ncsu.edu

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