

Kristin Umland

Illustrative Mathematics Co-chair
Dept. of Mathematics and Statistics
University of New Mexico
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COMMON CORE STATE STANDARDS FOR

Mathematics



Why a Common Core?

- 50 states
- 16,000 school districts

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- 16,000 school districts

There is no one at the helm of mathematics and science education in the U.S.... No single coherent vision of how to educate today's children dominates U.S. educational practice.... These splintered visions produce unfocused curricula and textbooks... [that] emphasize familiarity with many topics rather than concentrated attention to a few.... Our curricula, textbooks, and teaching are all "a mile wide and an inch deep."

-Schmidt, 1997

Grade Level Where Different States Introduce Addition and Subtraction of Fractions

Grade	Addition and Subtraction of Fractions
1 st grade	2 states
2 nd grade	
3 rd grade	7 states
4 th grade	22 states
5 th grade	9 states
6 th grade	1 state
7 th grade	1 state

Sources for the Common Core

- Standards of other countries and states
- Mathematics education research
- The structure of the subject
- National reports
- Commissioned research reviews and progressions
- Opinions of many groups and people

Structure of the Common Core

- Content standards by grade band K-8
- Content standards by “conceptual category” in HS
- Standards for mathematical practice woven throughout

Structure of the Common Core

- Content standards by grade band K-8
 - Each grade band has standards organized into a small number of domains

K	1	2	3	4	5	6	7	8
Geometry								
Measurement and Data					Statistics and Probability			
Number and Operations in Base Ten					The Number System			
Operations and Algebraic Thinking					Expressions and Equations			
Counting and Cardinality		Number and Operations--- Fractions			Ratios and Proportional Relationships		Functions	

Structure of the Common Core

- Each domain has standards organized into a small number of clusters

Operations and Algebraic Thinking

2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.¹

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.² By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Structure of the Common Core

- Content standards by “conceptual category” in HS



- Conceptual categories
 - Domains
 - Clusters
 - Standards

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Standards for Mathematical Practice

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8. Look for and express regularity in repeated reasoning.

- Not intended as free-floating proficiencies
- Not uniformly applied over all the work that students do

Design Principles for the Common Core

- **Focus**
- **Coherence**
- **Rigor**

Design Principles for the Common Core

- **Focus:** attending to fewer topics in greater depth at any given grade level, giving teachers and students time to complete that grade's learning.
- **Coherence**
- **Rigor**

Taking Focus Seriously

K	1	2	3	4	5	6	7	8
Geometry								
Measurement and Data					Statistics and Probability			
Number and Operations in Base Ten					The Number System			
Operations and Algebraic Thinking					Expressions and Equations			
Counting and Cardinality			Number and Operations--- Fractions			Ratios and Proportional Relationships		Functions

- For example: four out of the six domains in K-5 deal with numbers and operations. Many standards in the other domains support this focus.

The Payoff for Focus

- Taking focus seriously means delaying favored topics until their time, which will be a difficult shift for the educational system in the U.S.
- In high school, the subject matter focus broadens: algebra, functions, geometry, statistics, probability, modeling.
- Focus in high school means not so much a small number of topics as a concentration of skills and practice into a small number of underlying principles.

Design Principles for the Common Core

- **Focus**
- **Coherence:** attending to the structure of mathematics and the natural pathways through that structure, where “natural” means taking into account both the imperatives of logic and the imperatives of cognitive development in designing the sequence of ideas.
- **Rigor**

What is coherence?



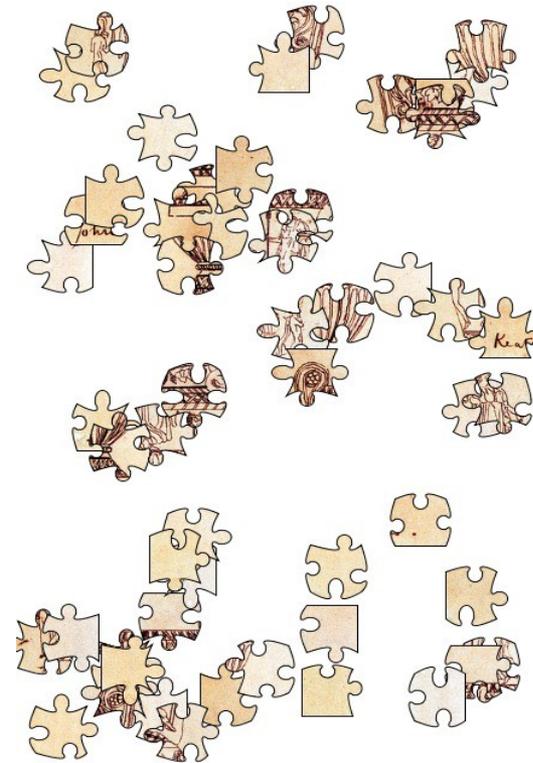
By John Keats.

A Grecian Urn

What is coherence?



A Grecian Urn



Standards for a Grecian Urn

Breaking with the Tradition of “Equal Grain Size”

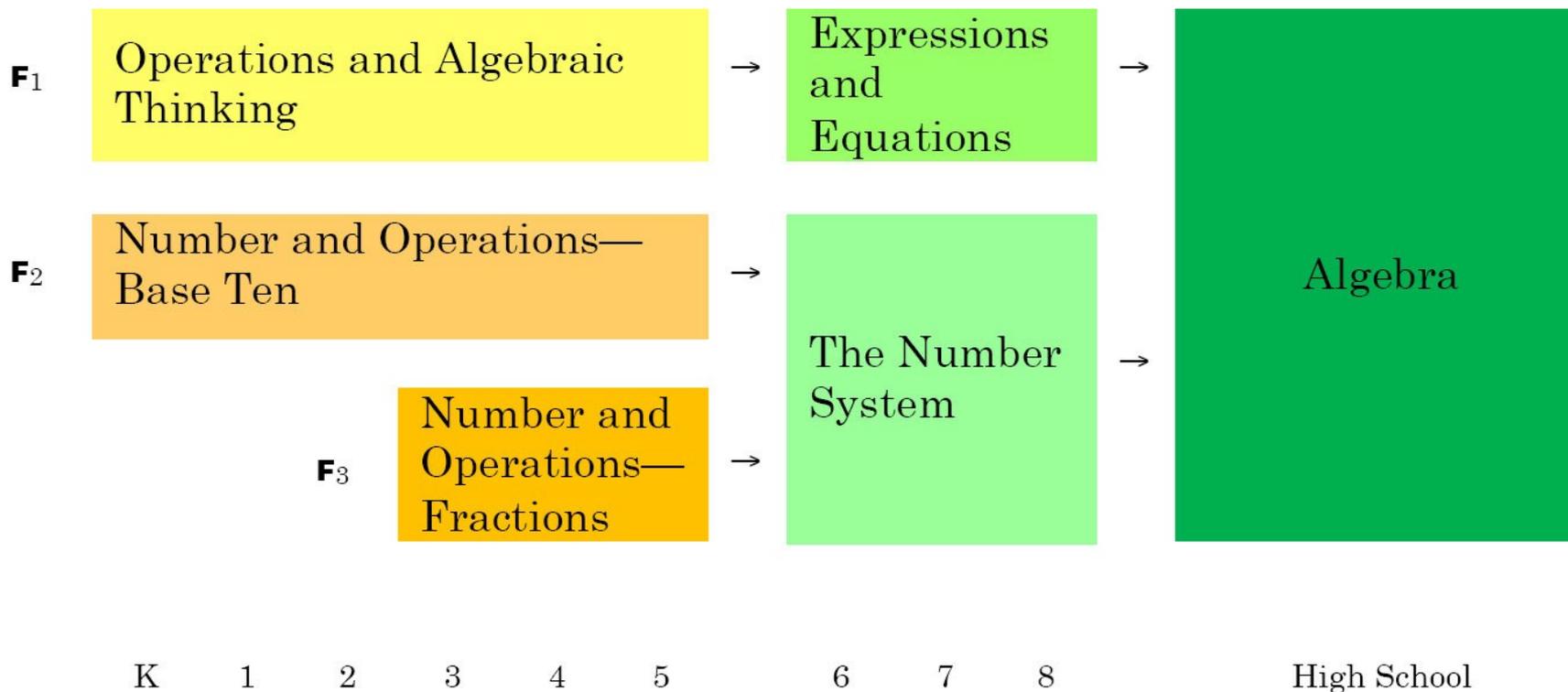
Number and Operations in Base Ten

2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
 - a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
 - b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Coherence Flows from Focus



Design Principles for the Common Core

- **Focus**
- **Coherence**
- **Rigor:** balancing conceptual understanding, procedural fluency, and meaningful applications of mathematics.

Conceptual Understanding

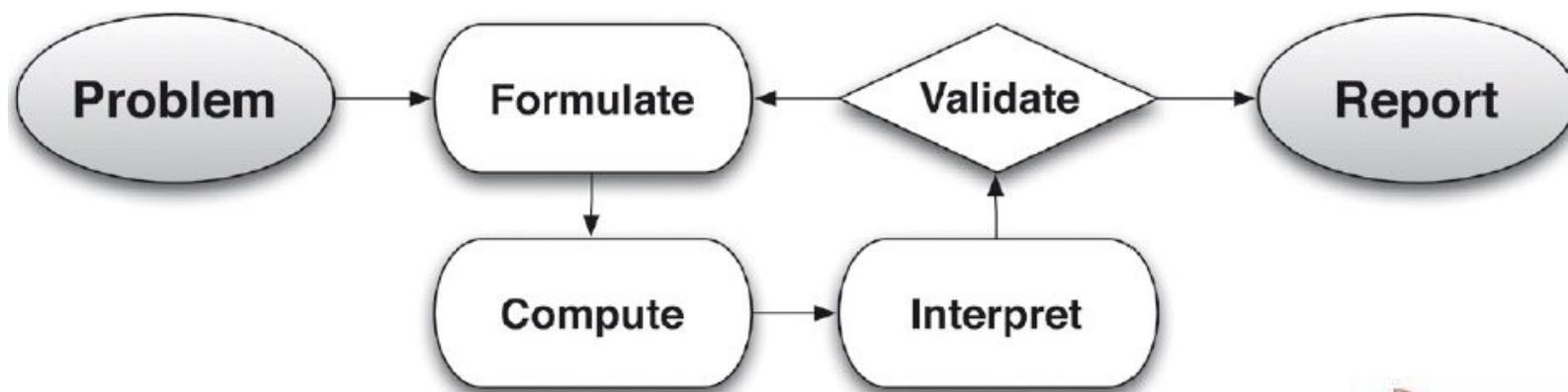
- Understand and apply properties of operations and the relationship between addition and subtraction (Grade 2)
- Understand concepts of area and relate area to multiplication and to addition (Grade 3)
- Understand ratio concepts and use ratio reasoning to solve problems (Grade 6)
- Understand congruence and similarity using physical models, transparencies, or geometry software (Grade 8)
- Understand solving equations as a process of reasoning and explain the reasoning (High School)
- Understand and evaluate random processes underlying statistical experiments (High School)

Procedural Fluency

- Single digit addition facts (Grade 2)
- Single digit multiplication facts (Grade 3)
- Addition and subtraction within 1000 (Grade 3)
- Addition and subtraction using standard algorithm (Grade 4)
(Note that addition and subtraction starts in Grade 1)
- Multiplication using standard algorithm (Grade 5)
(Note that multiplication and division starts in Grade 3)
- Division using standard algorithm (Grade 6)
- All operations on decimals using standard algorithm (Grade 6)
- Solving linear equations with rational coefficients (Grade 7)

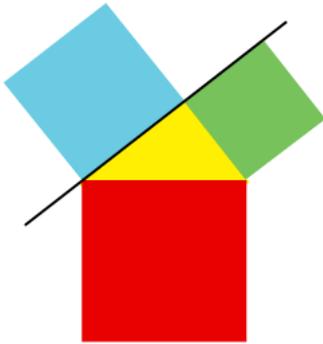
Meaningful Applications of Mathematics

- 7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- SMP.4. Model with Mathematics



For More Background/Info on the CCSSM

- *The Common Core State Standards for Mathematics* by William McCallum, presented at ICME 12, Seoul, Korea, July 2012 (the source for much of the first part of this talk (used with permission))
- Bill McCallum's blog "Tools for the Common Core Standards" <http://commoncoretools.me/>
- The Progressions Project for the CCSSM: <http://commoncoretools.me/category/progressions/>



Illustrativemathematics.org

Illustrative Mathematics is a community of math teachers, mathematicians, and mathematics educators. We provide leadership and guidance by illustrating the range and types of quality mathematics that students experience in a faithful implementation of the Common Core State Standards. We publish tasks and tools that support implementation of the standards.

Goals of Illustrative Mathematics

- Provide a resource for teachers implementing the CCSSM
- Provide guidance to states, assessment consortia, testing companies, and curriculum developers
- Create a discerning community of teachers, mathematics educators, and mathematicians who appreciate task writing as an art. Provide a forum for them to work together to develop and review tasks.



Who is involved in Illustrative Mathematics?

- Funded by the Bill and Melinda Gates Foundation
- Advisory board includes representatives of AMS, AMTE, ASA, ASSM, CCSSO, MAA, NCSM, NGA, NCTM, PARCC, Smarter Balance, etc.
- There are currently now almost 8,000 registered users (teachers, math educators, mathematicians)



What is a Task?

- A *mathematical task* is a problem or set of problems that focuses students' attention on a particular mathematical idea (content standards) and/or provides an opportunity to develop or use a particular mathematical habit of mind (MPs)
- Tasks can be used for *teaching* or *assessment*
- The mathematical idea or habit of mind that a task is intending to develop or assess, along with its intended use, the *purpose of the task*



Characteristics of a Good Task

- It has a well-defined purpose, and
- It has a reasonable chance of fulfilling its purpose if used appropriately.
- Defining the purpose of a task is a theoretical activity. Determining its efficacy is an empirical process.



7.NS.1 Baking Bread

A bread recipe calls for:

- $\frac{2}{3}$ cup milk
 - $\frac{1}{4}$ cup water
 - $1\frac{1}{2}$ cups whole wheat flour
 - 1 cups all-purpose flour
- a. Explain what the following sums mean, in the context of the recipe:
- $\frac{2}{3} + \frac{1}{4}$
 - $1\frac{1}{2} + 1$
- a. You want to combine all of the ingredients in a bowl. What is the minimum size (in cups) of the bowl that you need?



What Reviewers Had to Say

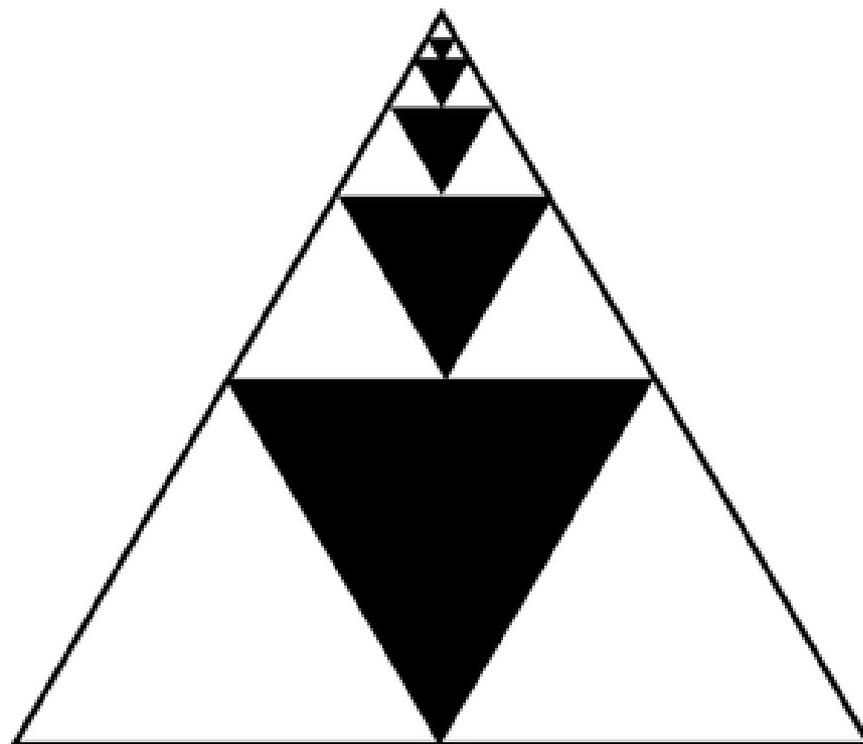
- The task is problematic on several levels:
 - It is limited to addition of fractions, which is covered by content standards in earlier grades, and does not extend understandings to the rational numbers.
 - The question regarding bowl size is completely irrelevant. A person making bread must choose a bowl that not only holds all the ingredients, but leaves ample room to beat them in a dough ready to be kneaded.



G-SRT.B.5 Triangle Series (version 1)

Find the total area of the black region given that the largest triangle in the diagram has area 1.

Commentary: The purpose of this task is to use similarity criteria to solve a geometric problem.



Illustrative Mathematics Task Review Form

Task criteria: Indicate whether the task meets each of the following criteria.

1. The task **illustrates** the specified standard, cluster, domain, or conceptual category.
2. The task's **purpose** is clearly stated in the commentary and is likely to be fulfilled.
3. The task has at least one appropriate **solution**.
4. The **mathematics** is correct.
5. Any diagrams or **pictures** have a clear mathematical or pedagogical purpose, which they are likely to fulfill.
6. The **context** supports the purpose of the task.
7. The task appropriately addresses **units and numerical precision**.
8. The language is **unambiguous** and grade-appropriate.



3.OA.9 Patterns in the Multiplication Table

Standard: 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*



The Future of Illustrative Mathematics

- Vision: Thousands (tens of thousands?) of teachers, math educators, and mathematicians working together
 - Lesson plans, well-developed rubrics and samples of student work, videos of students and teachers (all carefully reviewed)
 - Workspaces so people can modify tasks for their own use, put tasks together to form lessons, units
 - “Badge” system so people can get recognition for their contributions and expertise



Illustrative Mathematics

- Create and account at the website
- Comment on existing tasks
- Review tasks in development

<http://illustrativemathematics.org/>

- Contact us:

email@illustrativemathematics.org

